## 

@Unm; f]ZZ`Z]b]g\Yg`gYWcbX`]b`h\Y`8YfVm`Uh`9dgca ž \][\`][\h]b[`hfU]bYf`A Uf\_'&\bghcb/, \$' - /g`gi fdf]gY`Uh`h\Y cXXg`UbX`Zi hi fY`fUWY`dfcgdYWhg"



@Unm'@U\_Yž'I b]hYX'?]b[Xca'!'\b'h\Y'h\f]``]b['kcf`X'cZ'\cfgYfW\]b[z'h\Y'fY\Wbh'8\fVm'Uh'9dgca'g\ck\WgYX'h\Y'YI\W'dh]cbU`hU`Ybh'UbX'ghfUhY[m'h\Uh'XYZ]bYg'h\]g'\cadYh]h]j\Y'gdcfh'''H\Y\gdch`][\hik\Ug'cb'@Ua\Cifbž'U'\cfgY'hfU]bYX'Vm'h\Y'fYbck\bYX\s]XUb'C\6f]Ybž'k\c'gY\W'fYX'\]g'\%h\'8\YfVm'j\]\\cfm\z'\W'a\Ybh]b[\]g'ghUhi\g'\Ug'cbY'cZ'h\Y'[\fY\Uh\Ygh'hfU]b\Yfg']b'\]g\cfm\''@Ua\Cifb\g\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g'\df\g

@Unm; f]ZZžck bYX VmA ]XX`Y\Ua `DUf\_FUW]b[žZ]b]g\YX gYWcbXž

a Uf\_]b[ 'h\ Y 'gY Wc b X 'h]a Y 'h\ ]g'mY Uf'\ Y '\ Ug'VY Y b'ci hd UWY X 'V m @Ua Vci fb" 'H\ ]g'Xi c'U`gc'Zci b X 'h\ Ya gY`j Yg']b'h\ Y 'gUa Y 'dcg]h]cbg Xi f]b[ 'h\ Y '7 \ YghY f' J UgY '`Ugh'a cbh\ ž'k \ ]W\ 'cb`m'UX Xg'a cfY k Y][ \ h'hc'h\ Y]f'f]j U`fm" @Unm; f]ZZž'Ub'ci hg]XY f'Uh') \$! %z'k Ug Yl dY fh`m'\ Ub X`Y X 'V m' 7 \ f]ghcd\ Y 'Gci a ]``cb z'k \ c'bchY X 'h\ Uh XYgd]hY 'Wcb WY fbg'UV ci h'h\ Y 'hfUW\_g'[ fci b X 'Wcb X]h]cbgz'h\ Y '\ cfgY dY fZcfa Y X 'UXa ]fUV`m" = ZY`h' WcbZ]XY bh'Xi f]b[ 'h\ Y 'fUWY z'Vi h' = a ][ \ h'\ Uj Y 'a UXY 'U'hUWh]WU`a ]ghU\_Yz' Gci a ]``cb'fY ZY WhY X 'UZhY fh\ Y 'fUWY z'fYj YU`]b[ 'h\ Y 'Wca dY h]h]j Y 'bUhi fY 'cZ'h\ Y 'Yj Ybh"

## 5]XUb C 6f]Yb. 5 @Y[UWmcZGi WWYgg

5]XUb C 6f]Yb g fYa Uf\_UV Y 'ci fbYm'Ug'U 'hfU]bYf']g'k Y`
XcW a YbhYX"'G]bW' VYWca ]b[ 'h\Y'df]j UhY 'hfU]bYf'Uh'6U``mXcm`Y
GhUV 'Yg']b'%--\* ž'C 6f]Yb '\Ug'bch'cb`m'\cbYX'\]g'g\_]``g'Vi h'U`gc
Wcbh]bi ci g`mfU]gYX'h\Y'VUf'Zcf'YI WY``YbW'']b '\cfgY'fUW]b[ "'5g'\Y
UXXYX'mYh'Ubch\Yf'8YfVm'k ]b 'hc'\]g'UWWc`UXYgž'\]g'\cfgYg'\Uj Y
Wcbg]ghYbh`m'dYfZcfa YX'Uh'h\Y'\][ \Ygh'`Yj Y``UWfcgg'j Uf]ci g
dfYgh][ ]ci g'fUWYg"'C 6f]Yb'\Ug'hfU]bYX'bchUV`Y'ghUfg'`]\_Y'; U`]`Yc
UbX'7Ua Y`chž'UbX'\]g'UV]`]hm'hc'bi fhi fY'hU`Ybh']g'i bdUfU``Y`YX"
H\]g'fYWbh'k ]b'a Uf\_g'C 6f]Yb g'h\]fX'WcbgYW h]j Y'8YfVm'j ]Whcfmž
Zc``ck ]b[ i d'cb'\]g'CU\_g'hf]i a d\'k]h\'A]bb]Y'<Ui \_"

## $H\Y 7ca dYh]h]j Y : ]Y X$

5g'h\Y'8YfVm'i bZc`XYXž'h\Y'Z]Y`X'k Ug'Z]``YX'k ]h\'Ugd]fUh]cbg'UbXX]gUddc]bha Ybhg"'5a cb['h\Y'%, '\cfgYg'h\Uh'YbhYfYX'h\Y Wca dYh]h]cbž'h\Y'ZUj cf]hYž'8Y`UWfc]I ž'X]gUddc]bhYX'Vm'Z]b]g\]b[ b]bh\ž'UbX'H\Y'@|cb'+b'K]bhYf'WUa Y']b'%(h\"'H\Y'ghU\_Yg'k YfY \][\'Zc``ck]b['h\Y'k]h\XfUkU``cZ'&ž\$\$\$; i]bYUg'k]bbYf'Fi`]b[
7cifh'XiY'hc'WcbWYfbg'cjYf'h\Y'gcZhYbYX'[fcibX'WcbX]h]cbgž'k]h\d`Ubg'bck'hifb]b['hckUfXg'h\Y'Gh'\JaYg g'DU`UWY'GhU\_Yg'Uh'FcmU`5gWch"

- @Ua Vci fb. K ]bbYfcZh\Y8YfVmžhfU]bYXVm5]XUbC 6f]Yb
- @Unm'; f]ZZ ': ]b]g\YX'gYWcbXž'dfYj ]ci g`m'k cb'U'; fci d H\fYY
- HYbbYggYY'Ghi X.'7`U]a YX'h\]fX'i bXYf'ægYd\'C 6f]Yb g k UhW\Zi`'YmY
- 8 Y`UWfc]I . 'H\Y 'YUf`m'ZUj cf]hY 'Z]b]g\YX 'b]bh\

H\]g`mYUf g`8YfVm`U`gc`gUk `U`\YUfhk Ufa ]b[ `a ca Ybh`Ug`&gYd\C 6f]Yb`k UhW\YX`Zfca `h\Y`g]XY`]bYgž'dfci X`cZ`\]g`\cfgY HYbbYggYY`Ghi Xž`k\]W\`Z]b]g\YX`]b`h\]fX`d`UWY"`H\Y`Wca dYh]h]j Y gd]f]h`cZ`\cfgY`fUW]b[ `WfYUhYg`gi W\`bUffUh]j Ygž`Wca V]b]b[ `dUgg]cb k]h\`U`hci W\`cZ'dYfgcbU``ghcf]Yg`cZ'hf]i a d\`UbX`X]gUddc]bha Ybh"

cf@Unm; f]ZZUbX'hfU]bYf'A Uf\_'≈\bghcbžh\Y'dUh\'Zcfk UfX
fYa U]bg'cdYbžk ]h\'dchYbh]U``]bj ]hUh]cbg'Uk U]h]b[']b'h\Y'≢]g\
8 YfVm'cf'h\Y'; fUbX'Df]I 'XY'DUf]g"'H\Y'Ubh]W]dUh]cb'cZh\Y'bYI h
ci h]b[']g'dUfh'cZk\Uh'a U\_Yg'\cfgY'fUW]b['gc'Y`YWhf]W'A Uf\_Yh
Zi Whi Uh]cbg']b'VYhh]b['cXXg'WUi [\h'∞\bghcb'Vm'gi fdf]gYž
fYj YU`]b['`UmYfg'cZi bdfYX]WhUV]`]hm'h\Uh'W\UfUWhYf]nY'h\]g'gdcfh"
5g'ZUbg'`cc\_'U\YUXž'h\Y'dfca ]gY'cZ'a cfY'h\f]``]b['fUWYg'XfUk g
h\Ya 'cbž'k U]h]b['hc'gYY']Z'@Unm'; f]ZZ'WUb'Z]bU``m'[U]b'h\Y'i ddYf
\ccZ'U[U]bgh'@Ua Vci fb"

 $\label{thm:converse} 5g'h\ Y'gib'gYhg'cb'Ubch\ Yf'YlW]h]b['8YfVm'XUmž'cbY'h\ ]b[']g \ WYUf.'h\ Y'kcf'X'cZ'\ cfgY'fUW]b['Wcbh]biYg'hc'YbW\ Ubh'k]h\']hg \ V`YbX'cZ'g_]``ž'ghfUhY[mž'UbX'h\Y'YjYf!dfYgYbh'h\f]``'cZ \ WcadYh]h]cb"$ 

: cf'Zi fh\Yf'XYhU]`g'cb'h\Y'8YfVm'fYgi `hgž'j ]g]h'K Ygha cf`Ub X
; UnYhhY'cf'fYUX'a cfY'cb'MU\cc'Gdcfhg"'Hc'XY`j Y'XYYdYf']bhc
h\Y'`]ZY'UbX'UWX]Yj Ya Ybhg'cZ'5]XUb'C 6f]Ybž'WXYW\_'ci h'h\]g'
K ]\_]dYX]U'dU[Y"

8YhU]`g	
Cfh	@Unm`@U_Yž`I b]hYX`?]b[Xca
EiY``Yb	<ul><li>k k k "h\ Y k Y gha c f`Ub X [ Un Y hhY "Wc "i _</li></ul>
	<ul><li>gdcfhg"mU\cc"Wca</li><li>Yb"a "k ]_]dYX]U"cf[</li></ul>

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'a Y]b!k ]Yb "bYh