: GI '7cUWX Yg'9a VfUWY'B Yk '9fU. '5h\ `YhYg GYh'Zcf'F Yj Ybi Y'K]b X ZU``°

: GI / _ \$' - /g'A]WX UY``5`ZcfX'gi ddcfhg'bYk 'B 7 5 5'fYj Ybi Y!g\Uf]b[ž'[fUbh]b['Uh\ `YhYg'X]fYWh'dUma Ybhg'ghUfh]b['¬¬¬¬»½ &\$&) ž'dcghl`UbXa Uf_ gYhh`Ya Ybh"



I bXYf'h\]gʻgYhh`Ya Ybhžʻj U`i YXʻUhʻUʻk \cdd]b[ʻʻ &", ʻV]``]cbʻcj Yfh\YʻbYI hʻXYWUXYžʻ: `cf]XUʻGhUhYžʻUgʻUʻa Ya VYfʻcZʻh\Yʻ5h`Ubh]W 7cUghʻ7cbZYfYbWYʻfb77Łžʻk]```VYʻUV`YʻhcʻX]ghf]Vi hYʻi dʻhcʻ &\$") a]``]cbʻhcʻ]hgʻghi XYbh!Uh\`YhYgʻh\]gʻi dWca]b[ʻmYUf"'BchUV`mžʻ &")

a]``]cb cZ'h\]g 'Ua ci bh']g 'Y Ufa Uf_Y X 'Zcf g W c`Ufg\]d 'Z b Xgž 'U a cj Y 'h\ Uh 'gd Y U_g 'j c`i a Yg 'U V ci h': Gl g 'Wca a]ha Y bh'hc gi ddcfh]b[']hg 'Uh\ `Y h Yg 'V ch\ 'cb 'Ub X 'c ZZ'h\ Y 'Z]Y `X "' < ck Yj Y fž ``Y h g XY`j Y ']b hc 'h\ Y 'Wfi I 'c Z'h\ Y 'fYj Y bi Y 'g\ Uf]b[. 'Zcch V U```k]```fY W ']j Y U V ci h'+) ı 'c Z'h\ Y 'hch U``g\ UfYž a Y b g 'V Ug_Y h V U```Y gg 'h\ Ub '%) ı ž k]h\ 'k ca Y b g 'V Ug_Y h V U```Ub X 'U```ch\ Y f 'gdcfhg 'Wc``Y Wh]j Y `m'[Y hh]b[U a c X Y gh') ı 'c f '`Y gg z Ug 'f Y dcfh Y X 'V m' H U``U\ Ugg Y Y "Wca"

$H\Y 9bX cZ5a UhYi f]ga 3$

Hc h\]b_h\Uh^ gh'U'ZYk g\cfh'mYUfg'U[cžh\Y'B755 g'%-!mYUf!c`X Ua UhYi f]ga 'a cXY`ghccX'Z]fa `m']b'd`UWY"'6i h'k]h\ 1 X[Y'7`Ui X]U K]`_Yb g'Uddfcj U``cZh\Y'< ci gY'j "'B755'Ubh]hfi gh'gYhh`Ya Ybhž Yj Yfmh\]b['W\Ub[Yg"'H\]g'gYhh`Ya Ybh'k]``'Wca dYbgUhY'Uh\`YhYg Zfca '&\$%* cbk UfX'Zcf'a]ggYX'bUa Yž']a U[Yž'UbX'`]_YbYgg'fB = cddcfhi b]h]Yg"'I "G"'8]ghf]Wh' 1 X[Y'7`Ui X]U'K]`_Yb g'Uddfcj U` k fUdg'i d'Z]j Y'mYUfg'cZ'`]h][Uh]cb'UbX'bYUf`m'U'mYUf'cZ'dfcdcgYX X]gW gg]cbgž'a Uf_]b['U'g][b]Z]WUbh'hi fb]b['dc]bh'Zcf'Wc``Y[Y'gdcfhg Ug'bchYX'Vm'76GGdcfhg"Wca "

5gʻ]hʻghUb Xgž'YUWX ʻgWX cc``WUbʻbck ʻX]fYWh`mʻWca dYbgUhYʻUh\`YhYg Zcfʻh\Y]fʻWcbhf]Vi h]cbgžʻUbʻi bdfYWYXYbhYXʻa cj Yʻh\UhʻgYYa gʻhc cdYbʻUʻZccX[UhYʻZcfʻdchYbh]U``bYk ʻcddcfhi b]h]Yg"ʻ7i ffYbhʻWc``Y[YUh\`YhYgʻWUbʻYl dYWhʻhcʻgYYʻh\Y]fʻWca dYbgUh]cbʻbchʻʻl ghʻZcfʻcb! Z]Y`XʻdYfZcfa UbWYʻVi hʻUʻgcʻZcfʻh\Y]fʻcj YfU```VfUbXʻUgʻd`UmYfg"ʻH\Y]XYUʻ]gʻg]a d`YʻmYhʻdfcZci bX.ʻa U_Yʻgi fYʻh\Uhʻh\YgYʻmci b[Uh\`YhYgʻWUbʻdfcZ]hʻ]bʻUʻk Umʻh\UhʻfYZYWhgʻh\Y]fʻj Uʻi Yʻ]bʻh\Y

$B = 0.5[fYYa Ybhg UbX : i hi fY = bg][\hg$

H\]grgYhh`Ya Ybh`XcYgb h`^l gh'hci W\`cb`fYj Ybi Y`g\Uf]b[/]h`U`gc UZYWhg`h\Y``cb[!ghUbX]b[`Wcbj YfgUh]cbg`Ufci bX`B @`U[fYYa Ybhg" GhUfh]b[`¬¬¬``Y[Y`Gdcfhg`7ca a]gg]cb`k]```cj YfgYY Wcbhf]Vi h]cbg`UbX`a cb]hcf`B@`XYU`g Ybgi f]b[`h\Uh`Uh\`YhYg fYWY]j Y`ZU]f`a Uf_Yh'j U`i Y`Zcf`h\Y]f`YbXcfgYa Ybhg`k\]`Y`gW\cc`g WUb`Ugg][b`VYhk YYb``) ž\$\$\$`UbX``) \$\$ž\$\$\$ Zcf`hfUW_]b[`gcZhk UfY XYj Y`cdYX`Vm`8Y`c]hhY"`H\]g`]g`Ub`]a dcfhUbh`ghYd`Zcfk UfXž

 $\label{thm:cgyk} YgdYWJU``m`Zcf`h\cgY`k \c`Zci [\h`\UfX`Zcf`h\Y]f`f][\hg`k]h\]b`h\Y gmghYa "$

6i h'Wk U``Yb [Yg``cca "'K \]`Y'h\Y']a a YX]UhY'Zi hi fY``cc_g'Vf][\h'Zcf d`UmYfgž'h\Y'Z]bUbW]U``Vi fXYb'cb'i b]j Yfg]h]Yg'Wci `X``YUX'hc W'h\VU_g']b'ch\Yf'UfYUgž'Zcf\]b['h\Ya 'hc'fYUggYgg'gdYbX]b['UbX fYgci f\Wg''K \Uh'\UddYbg'bYI h3'H\YfY g'd`Ybhm'cZ'fcca 'Zcf'Zi hi fY X]g\W gg]cbgž'YgdY\]U``m'fY[UfX]b['H]h`Y'\\\\\ '\&a d`]Ub\W'`UbX Yei]hUV`Y'fYj Ybi Y'X]ghf]\Vi h]cb'Ua cb['[YbXYfg''I bZcfhi bUhY`mž]h g'k cfh\'bch]b['h\Uh']bhYfbUh]cbU``ghi XYbh!Uh\`YhYg'a][\h'ZU\W\\ifX`Yg'k]h\'B\\\\@\\Ca dYbgUh]cb'Xi Y'hc'fYghf]\\\h]j Yj]gU fY[i`Uh]cbg gca Yh\]b['h\Uh'\\U'`g'Zcf'i f[Ybh'UhhYbh]cb''

; fUbXZUh\YfYX'D`UmYfg'UbX'FcghYf A UbU[Ya Ybh

 $\label{thm:spanning} 5bch\ Yf'_Ym'dc]bh']g'\ ck'h\ Y'gYhh'Ya\ Ybh'UZZYWhg'fcghYf'`]a\]hg'' : `cf]XU'GhUhY'W'ffYbh`m'\ Ug'%$, 'd`UmYfgž'YI WYX]b['h\ Y'bYk'']a\]hž Vi h'h\ YfY g'U'g]`j Yf'`]b]b['''H\ Y'gYhh`Ya\ Ybh'U``ck g'Zcf'd`UmYf [fUbXZUh\ Yf]b['z'YbUV`]b['h\ cgY'U`fYUXm']b'h\ Y'gmghYa''hc Wca d`YhY'h\ Y]f'Y`][]V]`]hm'k]h\ ci h'h\ Y'`cca]b['h\ fYUh'cZ'W' hg'Zfca h\ Y'fcghYf'''<ck Yj Yfz'`cc_]b['U\ YUXz'ZcchVU``'fcghYfg'k]``VY WUddYX'Uh'%$)''d`UmYfg'Ug'h\ Y'B755'U]a\ g'hc'ghfYUa\ `]bY'h\ Y\ bi a\ VYfg'k\]`Y'UWWca\ a\ cXUh]b['YI]gh]b['hU`Ybh''$

H\]g']g'hfi `m'U'bYk 'W\UdhYf'Zcf'Wc``Y[Y'gdcfhg U'h]a Y'cZ Wcbg]XYfUV`Y'W\Ub[Y'UbX'Ub'cddcfhi b]hm'Zcf'Uh\`YhYg'hc WUd]hU`]nY'cb'h\Y]f'hU`Ybhg'UbX'YZZcfhg'`]_Y'bYj Yf'VYZcfY"'H\Y ei Ygh]cb'bck 'fYa U]bg.'<ck 'k]``'h\YgY'W\Ub[Yg'fYg\UdY'h\Y Zi hi fY'cZ'Wc``Y[Y'Uh\`Yh]Wg'UbX'h\Y'Yl dYf]YbWYg'cZ'a]``]cbg'cZghi XYbh!Uh\`YhYg3

8 YhU]`g	
Cfh	5`ZcfXž'lb]hYX'GhUhYg
EiY``Yb	• k k k "hU``U\UggYY"Wca
	kkk"VWggdcfhg"Vca
	h\YWcbjYfgUh]cb"Wca

6Ygi WXYb G]Y i bg Ui Z a Y]b!k]Yb"bYh