7\Ufa]b['?]hhYb`FYgW'YX`8if]b['6cb]hU Gdf]b[g'HfUZZ]WGhcd.`AYYh`6Uffm°

9| d`cfY`fYWYbh`Yj Ybhg`]b`@YY`7ci bhmž`Zfca `U`\YUfhk Ufa]b[__]hhYb`fYgW'Y`Xi f]b[`U`hfUZZ]Wghcd`hc`gYf]ci g`Ub]a U``Wfi Y`hm WUgYg"



6cb]hU'Gdf]b[gž: `cf]XUžI G5'!-\b'U'\YUfhk Ufa]b[a ca Ybh h\Uh'\Ug'fYgcbUhYX'k]h\a UbmfYg]XYbhgž'U'hfUZ]Wghcd']b'6cb]hU Gdf]b[gž: `cf]XUžhi fbYX']bhc'Ub'i bYl dYWhYX'fYgW Y'cdYfUh]cb'cb \(bY' ("'6cXmWUa 'ZcchU[Y'Zfca 'h\Y'@YY'7ci bhm'G\Yf]ZZ g'C ZZ]WW WUdhi fYX'h\Y']bW]XYbhž'k\YfY'U'ghfUm'_]hhYb'a UXY'Ub'UXcfUV'Y UddYUfUbW"'H\Y'Xf]j Yf'\i a cfci g`m'Xi VVYX']h' h\Y'VYgh'di ``!cj Yf Yj Yf'' :: cl '%' 'BYk g'fYdcfhg'h\Uh'h\Y'Xf]j Yfž'YU[Yf'hc'\Y'd'h\Y a Yck]b['_]hhYbž'gci [\h'dYfa]gg]cb'Zfca 'h\Y'XYdi hm'hc'Ugg]gh'' 5``ck]b['h\Y'Xf]j Yf'hc'Yl]h'h\Y'j Y\]WYž'h\Y'XYdi hm'hcc_'h\Y'_]hhYb]bhc'\]g'Wfi]gYfž'`][\h\Y'UfhYX`m'Wca a Ybh]b[ž' H\Y'WUh'XYW]XYX mci fY'[Yhh]b['U'k Ufb]b['" 'H\Y'_]hhYbž'bck 'UZZYWh]cbUhY`m'bUa YX 6Uffmž']g'Uj U]`UV'Y'Zcf'UXcdh]cb'h\fci [\'@YY'7ci bhm'5b]a U` GYfj]WYg''

Gi W. \YUfhk Ufa]b[]bWXYbhg g\]bY U ``][\h'cb h\Y]a dcfhUbWY cZ $\label{thm:linear} Ub]a\ U``k\ Y`ZUfY']b``ci\ f``Wca\ a\ i\ b]h]Yg'' I\ b\ Zcfhi\ b\ UhY\ `mz`bch``U```ghcf]Yg$ $g\UfY'h\g'^cm''5'[\Uf]b[\WcbhfUgh'Ya\Yf[\YX'k\Yb'5bh\cbm'?b][\h$ $Zfca @Y \ [\ \ 5 Wf Yg ZUWY X gYf]ci g Y[\ U Yf Yd Yf W gg]cbg UZh Yf$ $dcgh]b["cb": UWYVcc_"UVcih"h\Y"hfU[]WXYUh\"cZ"\]g"Xc[""<]g$ X]ga]gg]j Y Wca a Ybhgž]bWi X]b['U'gi [[Ygh]cb'h\ Uh'h\ Y '`cgg'k Ug a YfY`m' `%\$ž\$\$\$'Xck b'h\Y'XfU]bž 'XfYk'h\Y'UhhYbh]cb'cZ WcbWYfbYX:WjhjnYbg:UbX:`Uk:'YbZcfWYaYbh":H\Y:@YY:7cibhm ?b][\hg:Xc[ž:?U`YUž:\UX:VYYb:`YZh:]b:X]qhfYqq]b[:WcbX]h]cbqž i`h]a UhY`m'`YUX]b['hc'\Yf'XYa]gY']b'U'WU[Y'k]h\cih'kUhYf'cf j Ybh]`Uh]cb"'5g']h'ghUbXqž'?b][\h'k Ug'W\Uf[YX'k]h\'ZY`cbm'Ub]a U` $Wfi Y hmž fYj YU b[hhy XUf_Yf g]XY c Z Ub]a U ck b Yfg d" Hhg WUgY$ i bXYfgWcfYg'h\Y'Wf]h]WU``bYYX'Zcf'[fYUhYf'Uk UfYbYgg'UbX j][]`UbWY`U[U]bgh`Ub]a U``WfiY`hm`]b`cif`Wcaaib]h]Yg"`<mark>BYkgkYY_</mark> Y`UVcfUhYg`cb`\ck': `cf]XU``Uk'XYZ|bYg`Ub]a U``Wfi Y`hm`UbX $\[\] \] \$ hg h\ Y XUb [Yfg c Z \ YUh YI dcgi fY Zcf Ub]a U g"

I bXYfghUbX]b[': `cf]XU g'5b]a U`'K Y`ZUfY @Uk g

#b`fYWYbh`mYUfgž:`cf]XU`\UgʻhU_Yb`g][b]Z]WUbh`ghf]XYgʻ]b`fY[i`Uh]b[Ub]a U``k Y`ZUfY`h\fci [\`Wca dfY\Ybg]j Y``Y[]g`Uh]cb"`5g
XcWa YbhYX`Vm`GhUhYFY[g`HcXUmžh\Y:`cf]XU`5b]a U``FYgW'Y
5Wh`fl 5F5Ła UbXUhYgʻh\Uh`U```Ub]a U``g\Y`hYfgʻUbX`fYgW'YgʻVY
`]WYbgYX`UbX`UX\YfY`hc`ghf]Wh`WUfY`ghUbXUfXg"`Gi W\`a YUgi fYg
]bWi XY`cV`][Uhcfm'j UVW]bUh]cbgʻUbX`a YX]WU``hfYUha Ybh`VYZcfY
UXcdh]cbžk]h\`gYj YfY`WcbgYei YbWYg``]_Y`Z]bYgʻcf``]WYbgY
gi gdYbg]cb`Zcf`bcb!Wca d`]UbWY"::ifh\Yfa cfYž'bc`g\Y`hYf`WUb
UXcdh'ci h`Ub]a U`g`k]h\ci h`Z]fgh`WcbXi Wh]b[`gdUm`UbX`bYi hYf
cdYfUh]cbgž'Zifh\Yf`Wcbhf]Vi h]b[`hc`fYgdcbg]V`Y`dYh`ck bYfg\]d
UbX`fYXi W]b[`cj YfWfck X]b[`]b`g\Y`hYfq"

H\Y`cb[c]b[`VUhh`Y`U[U]bgh`cj YfWfck X]b[`UbX`Yi h\UbUg]U`]g gi ddcfhYX`Vm`ci hfYUW\`]b]h]Uh]j Ygž`dUfhbYfg\]dg`k]h\`fYgW`Y cf[Ub]nUh]cbgž`UbX`Wca a i b]hm`Yb[U[Ya Ybh`hc`dfca chY`gdUm]b[UbX`bYi hYf]b["`K]h\`h\Y`YbZcfWYa Ybh`cZ`h\Y` Bc!?]`` `dc`]Wm YghUV`]g\YX`Vm`h\Y`8c[`FYgW`Y`6]``g`]b`&\$%(ž:`cf]XU`Wcbh]bi Yg`hc df]cf]h]nY 'h\Y '`]j Yg'cZ'g\Y`hYf'Ub]a U`g"'FY[i`Uf']bgdYWh]cbg'Ybgi fY Wca d`]UbW'k]h\ 'ghUhY 'fY[i`Uh]cbgž'UbX 'hfUbgdUfYbWm']g Ya d\Ug]nYX 'h\fci [\'di V`]WX]gWcgi fYg'UbX 'Ubbi U`'fYdcfh]b[fYei]fYa Ybhg"'Gi W\ 'fY[i`Uh]cbg'd`Um'U'Wfi W]U`'fc`Y']b'Ybgi f]b[h\Uh'Yj Yfm'Ub]a U`'\Ug'U'W\UbW''Uh'Z]bX]b['U'`cj]b['\ca Yž'ai W\`]_Y'6Uffm'h\Y'_]hhYbž'UbX 'fU]g]b['Uk UfYbYgg'UVci h'\ia UbYhfYUha Ybh']b'Vch\'g\Y`hYfg'UbX'df]j UhY'\ca Yg"

Ghcf]YgʻZfca 'Vch\ 'YbXgʻcZʻh\ YʻUb]a U``WUfYʻgdYWhfi a 'fYa]bXʻi g h\ Uhʻk \]`Yʻh\ YfYʻUfYʻa ca YbhgʻcZʻʻcmʻUbXʻWca dUgg]cbžʻh\ YfYʻUfY UʻgcʻdfYgg]b[']ggi Ygʻk YʻWUbbch'][bcfY"ʻ5gʻWca a i b]hmʻa Ya VYfgž]h gʻj]hU``k Y`UXj cWUhYʻZcfʻUb]a U``k Y`ZUfYžʻfYdcfhʻgi gdYWhYX Wfi Y`hmžʻUbXʻgi ddcfhʻ`cWU``g\ Y`hYfgʻ]bʻh\ Y]fʻa]gg]cbʻhcʻdfchYWhʻUbX WUfYʻZcfʻU```WfYUhi fYg"ʻ9UWXʻga U```ghYdʻk YʻhU_YʻWUbʻWcbhf]Vi hYʻhc WfYUh]b['UʻgUZYfʻUbXʻa cfYʻWca dUgg]cbUhYʻYbj]fcba YbhʻZcf Ub]a U`gʻh\ fci [\ci h: `cf]XU"

8 Y hU]`g	
Cfh	6cb]hU'Gdf]b[gž': `cf]XUž'l G5
EiY``Yb	• k k k "Zcl %' b Y k g"Wca
	• k k k "b Y k g k Y Y _ "Wca
	• k k k "ghUhYfY[ghcXUm"Wca

6Ygi WXYb G]Y i bg Ui Z. a Y]b!k]Yb bYh