: `cf]XU'7]hfi g'; fck Yfg'GhUbX':]fa .'<cdY 5a]X'<]ghcf]W7\U``Yb[Yg

8 UXY '7]hmž: `cf]XUžI G5 !!: `cf]XU g'W]hfi g']bXi ghfm`\Ug'gYYb VYhhYf'XUmg/\ck Yj Yfž'U'[`]a a Yf'cZ\cdY'Wcbh]bi Yg'hc'g\]bY h\fci [\'h\Y'W\U``Yb[Yg'ZUWYX'Vm'[fck Yfg"'Cj Yf'h\Y'mYUfgž'h\Y]bXi ghfm`\Ug'VYYb'VUhhYfYX'Vm'W]a Uh]WUXj Yfg]h]Yg'UbX'X]gYUgYgž mYh'fYg]`]YbWY'h\f]j Yg']b'h\Y'\YUfhg'cZ'a Ubm'k \c'UfY'XYX]WUhYX'hcfYj]j]b['h\]g'j]hU``gYWhcf'cZ': `cf]XU g'YWcbca m"'5g'BYk g!DfYgg dc]bhg'ci hž'XYgd]hY'U'`cb['\]ghcfm'cZ'gYhVUW_gž']bWi X]b['h\YXYj UghUh]b['ZfYYnYg'cZ'h\Y'%-, \$gž'cdh]a]ga 'fYa U]bg'U`]j Y'Ua cb[h\Y'[fck Yfg"'

H\Y`dYW``]Uf`ZfYYnY`cb`7\f]gha Ug`8Um`]b`%-,'```YZh`U`Zfcghm
]a df]bhž`Xfcdd]b[`[fcj Y`dfcdYfhmj U`i Yg`XfUa Uh]\WU``m`Zfca
`&ž\$') `dYf`U\WfY`hc`` *&\$`cj Yfb][\h"`Gi W\`Zi \hi \hi \h]cbg``YX`hc`U
hfci V`]b[`XYW]bY`]b`dfcXi \h]cbž`k\YfY`\M]hfi g`ci hdi h`ZY```Zfca `&' \$
a]``]cb`Vcl Yg`]b`%-, \$`hc'%) %`a]``]cb`Vcl Yg`Vm'%--\$ U
ghU[[Yf]b[`Xfcd`cZ''(ı "`Cb`U`Vf][\hYf`bchYž'%--, `gUk
dfcXi \hM]cb`Vci b\W`VU\M_`hc`Ub`]a dfYgg]j Y'' \$\%`a]``]cb`Vcl Ygž
g\ck \hUg]b[`h\Y`]b\Xi ghfmg`\WUdU\M]hm`hc`fYVci b\X''<ck Yj Yfž'h\]g
k ci `X`bch`VY`h\Y``Ugh`\W\U``Yb[Yž`Ug`Yd]XYa]\\g`cZ`\M]hfi g`\WUb_Yf
Ub\Xž'a cfY`fY\W\bh`mž\M]hfi g`[fYYb]b[`\Uj Y`a UffYX`h\Y``Ub\Xg\WUdY
cZ:`cf]XU g`\M]hfi g`Z]Y`\Xg"

$H\Y @UhYgh 7\U``Yb[Yg$

 $\label{eq:hamiltonian} H\Y`g]hi\Uh]cb`\Ug`bch`]a\dfcj\YX`g][\b]Z]WUbh`m"`5`8YWYa\VYf`fYdcfh$

Vm: `cf]XU'HUI K UhW\ XYhU]`YX'h\Y'cb[c]b['XYW]bYž'fYj YU`]b['U g\cW_]b['XYWfYUgY'cZ'cj Yf'-\$i ']b'hchU`'W]hfi g'dfcXi Wh]cb'cj Yf h\Y'dUgh'hk c'XYWUXYg Zfca 'UddfcI]a UhY`m'' \$\$'a]``]cb'VcI Yg']b h\Y'&\$\$' \$('gYUgcb'hc'U'a YfY'&\$'a]``]cb']b'&\$&' &('"'H\Y'fYdcfh Ya d\Ug]nYX'h\Uh'h\Y'&\$&& &' 'gYUgcb'k Ug'h\Y'k cfgh'Zcf'h\Y]bXi ghfm']b'bYUf`m'U'WYbhi fm''

7]hfi g'[fYYb]b['X]gYUgYž'U`gc'_bck b'Ug'<i Ub[`cb[V]b['fk@6\Z'\Ug'VYYb'Uh'h\Y'WbhYf'cZ'h\YgY'hfci V`Yg'g]bW']hg']XYbh]Z]WUh]cb
]b'&\$\$) "'5g'7]hfi g'=bXi ghfm'bchYgž'h\]g'X]gYUgY'g][b]Z]WUbh`m
UZZYWhg'h\Y'j UgW'`Uf'gmghYa 'cZ'W]hfi g'hfYYgž'`]a]h]b['bi hf]Ybhg'UbX
Xf]j]b['dfcXi Wh]cb'Wcghg'\][\Yf"'=hg'U'[f]a 'fYU']hm'k\YfY'a Ubm
[fck Yfg'ZYY`'h\Y'd]bW\/f]g]b['dfcXi Wh]cb'Wcghg'UbX'W\U``Yb[]b[
[fck]b['WcbX]h]cbg'UfY'di g\]b['gca Y'ci h'cZ'Vi g]bYggž'ZcfW]b[
h\Ya 'hc'gY``'h\Y]f'`UbXg'hc'XYj Y`cdYfg"'

:]bX]b['U'K Um': cfk UfX

MY hž Ugʻk YʻWY`YVfUhYʻh\YʻfYg]`]YbWYʻc Zʻh\]gʻWca a i b]hmžʻ]bhYfj]Yk g WcbXi WhYX`k]h\`W]hfi gʻ[fck YfgʻfYj YU`ʻh\Uh'' (ʻci hʻc Z''*ʻfYa U]b\cdYZi``UVci hʻh\Yʻ]bXi ghfm gʻZi hi fY"ʻE i Ybh]bʻF c YžʻdfYg]XYbhʻc Z K "; "'F c Y'/ 'GcbgžʻYl dfYggYX'df]XY`]bʻh\YʻfYg]`]YbWYʻc Zʻ\]g Wca dUbm'' '\&\b'6UfVYb`\][\`][\`][\hYX`VfYU_h\fci [\ihfYUha YbhgʻZcf W]hfi gʻ[fYYb]b[ž']bgh]``]b['UʻgYbgYʻc Zʻcdh]a]ga 'Ua]X'hf]Vi`Uh]cb"'

a $Uf_Yhd^UWY^XYgd]hY^hVY^XYW]b]b[^dfcXi Wh]cb^fUhYg^*$

 $\label{lem:decomposition} Dfca \]g]b[\ YZZcfhg`UfY`ibXYfk Umž`k \]h\ `fYgYUfWXYfg`YId`cf]b[\]bbcj Uh]j Y`gc`ih]cbg``]_Y`$\DfchYWh]j \ Y`GWfYYbg`f\D7g\ZcfbifgYfm`hfYYg`UbX`hfib_`]bYWh]cbg`k \]h\ `cImhYhfUWmW]bY \ mXfcWX`cf]XY`fCH7\L''\5XX]h]cbU``mž`UXj UbWYX`7F \ GDF`hYWXbc`c[m \]g`VY]b[``Udd`]YX`hc`XYj Y`cd`X]gYUgY!fYg]ghUbh`hfYYgž`h\ci[\`h\YkU]h`Zcf`hUb[]V`Y`fYgi`hg`Wcbh]bi Yg''`$

5g: `cf]XU g'W]hfi g']bXi ghfm'ghUbXg'Uh'U'WfcggfcUXgž'cbY'WUbbch cj Yf`cc_'h\Y'i bk Uj Yf]b['XYhYfa]bUh]cb'cZ']hg'dYcd`Y''8Ygd]hY h\Y'a Ub]Zc`X'W\U``Yb[Ygž'h\Y]f'dUgg]cb'Zcf'dfcXi W]b['ei U`]hm W]hfi g'YW\cYg'h\Y'gYbh]a Ybh 'k \YfY'h\YfY']g'U'k]``ž'h\YfY']g'U k Um"' H\YfY g'gca Yh\]b['hc'VY'gU]X'Zcf'h\Y'fYg]`]YbW'cZ': `cf]XU g [fck Yfgž 'UbX'Ug'h\Ym'Wcbh]bi Y'g\Ud]b['h\Y'Z hi fY'cZ'W]hfi gž'k Y k]```VY'f][\h'VYg]XY'h\Ya ž'W\YYf]b['h\Ya 'cb"

8 YhU]`g	
Cfh	8UXY ⁻⁷]hmž ⁻ : `cf]XUž ⁻ I G5
EiY``Yb	• k k k "b Y k g! d f Y gg" Wca
	W]hfi g]bXi ghfm"bYh
	• YX]g"]ZUg"i Z`"YXi

6Ygi WXYb 'G]Y 'i bg 'Ui Z. 'a Y]b!k]Yb "bYh